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PENETRATION OF AN EXTERNAL FIELD CHANGE WITH ARBITRARY ANGLE INTO A SATURATED SUPERCONDUCTING FILAMENT

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Abstract

In many applications of superconductivity the magnetization of a saturated cylindrical superconducting filament is perturbed by an external field change. Especially in superconducting accelerator magnets local field changes have an impact on the magnetization in the coil. We analyze the behavior of the current distribution and magnetization of a single filament for field changes with different amplitudes and at arbitrary angles with respect to the original background field. Formulas are developed and the behavior of the magnetization is demonstrated.

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Penetration of an external field change with arbitrary angle into a saturated superconducting filament

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Abstract

In many applications of superconductivity the magnetization of a saturated cylindrical superconducting filament is perturbed by an external field change. Especially in superconducting accelerator magnets local field changes have an impact on the magnetization in the coil. We analyze the behavior of the current distribution and magnetization of a single filament for field changes with different amplitudes and at arbitrary angles with respect to the original background field. Formulas are developed and the behavior of the magnetization is demonstrated.

Keywords: Filament, Magnetization, Penetration, Superconductor

1. *Introduction*

In this paper the impact of an external field change with arbitrary direction on the magnetization of a saturated (type II) superconducting filament is analyzed. The superconductor is modeled using an algorithm by Brandt [1]. The behavior of the current distribution is shown and formulas are developed to describe the magnetization change. The topic has several applications in superconductivity. Especially in superconducting accelerator magnets local field changes have an impact on the filament magnetization in the coil. Previous work on related topics was presented by Pang [2], Rem [3] and Hartmann [4].

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1.1. Geometry of the Problem

A cylindrical filament with radius R is oriented parallel to the z-axis of a coordinate system. The center of the filament cross section is placed into the origin. Perpendicular to the filament axis a magnetic field is applied. The penetration field B_P and the saturation magnetization M_0 are given for the case of a superconductor in the critical state [5] with a constant critical current density j_C [4]:

$$B_P = \frac{2m_0 j_C R}{p} \tag{1}$$

and

$$M_Y = M_0 = \frac{4m_0}{3p} j_C R. \tag{2}$$

We assume a background field $\boldsymbol{B}_0 = -B_0 \boldsymbol{e}_y$ with B_0 equal or greater than B_P. A field change $D\boldsymbol{B}=-(B_X, B_Y)$ is applied at an angle j with respect to $\boldsymbol{B}_0$.

1.2. Symmetries

For a homogeneous background field and a filament without transport current the shielding currents are antisymmetric with respect to the origin: $J(x, y) = -J(-x, -y)$. As a consequence, the magnetization after the field sweep also has symmetries:

$$M_X(j) = -M_X(2p - j) \quad (3)$$

and

$$M_Y(j) = M_Y(2p - j)\,. \quad (4)$$

Thus, the problem can be restricted to $0° \le j \le 180°$.

1.3. Formulas

For $j = 0°$ we find M_X=0 and M_Y= M_0. Also at $j = 180°$, M_X equals 0 and for $|DB| = 2B_P$ an analytical approximation of M_Y/M_0 [4] is

$$M_Y/M_0 = \left(1 - \left(1 - (\Delta B/2B_P)\right)^3\right). \quad (5)$$

The shielding currents initially present in the filament have to change their sign and only saturate after a field sweep of $2B_P$. In order to take this effect into account we will refer to the reduced field change Db=$|DB|/2B_P$. The magnetization for field sweeps with arbitrary j and Db»1 saturates so that

$$\boldsymbol{M}_{Sat} = M_0(\sin j, \cos j). \quad (6)$$

A reasonable normalization for the magnetization components is:

$$m_X(j, \Delta b) = \frac{M_X(\Delta B)}{M_0 \sin j} \quad (7)$$

and

$$m_Y(j, \Delta b) = \left(1 - \frac{M_Y(\Delta B)}{M_0}\right) \Big/ (1 - \cos j). \quad (8)$$

We use the third order approximation in Eq. (5) as a reference curve and subtract it from the normalized magnetization in Eq. (7) and (8):

$$f_X(j, \Delta b) = m_X(j, \Delta b) - 1 + (1 - \Delta b)^3 \quad (9)$$

and

$$f_Y(j, \Delta b) = m_Y(j, \Delta b) - 1 + (1 - \Delta b)^3. \quad (10)$$

The 'error functions' f_X and f_Y describe the deviation of m_X and m_Y from Eq. (5). In order to fit f_X and f_Y the following expression is used:

$$f(j, \Delta b) = A(\Delta b/b_S)^a \left(1 - (\Delta b/b_S)\right)^b \left((\Delta b_0 - \Delta b)/b_S\right)^g. \quad (11)$$

The exponents a, b, g, the amplitude A, the saturation field $b_S = B_S/(2B_P)$ and the zero position Db_0 are functions of j and can be adapted to calculated data.

2. RESULTS

2.1. Simulations

In order to reduce the problem to its basic mechanisms, the critical current density j_C is taken as a constant and independent of the magnetic field. We approximate the critical state model [5] and the case of no flux flow, using a high creep exponent of 500. We assume a typical LHC filament with R=3.5µm and j_C=1.058·10^{10} A/m^2. Its cross section is meshed with a rectangular grid. Field changes are applied in angular intervals of 5°, between 0° and 180° with respect to the initial background field of 0.5 T. In each case DB is increased from 0 to 0.3 T. In Fig.

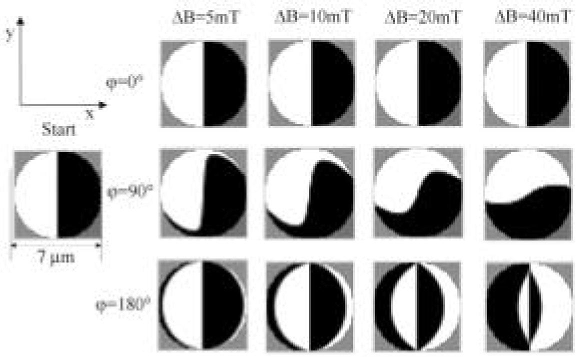

Fig. 1. The distribution of shielding currents in the filament cross section is shown for different field changes at angles j of 0°, 90° and 180°.

1 the impact on the shielding currents in the superconductor is demonstrated for angles j of 0°, 90° and 180°. M_X, M_Y, m_X, m_Y, f_X and f_Y are plotted in Fig. 2 as functions of j and DB.

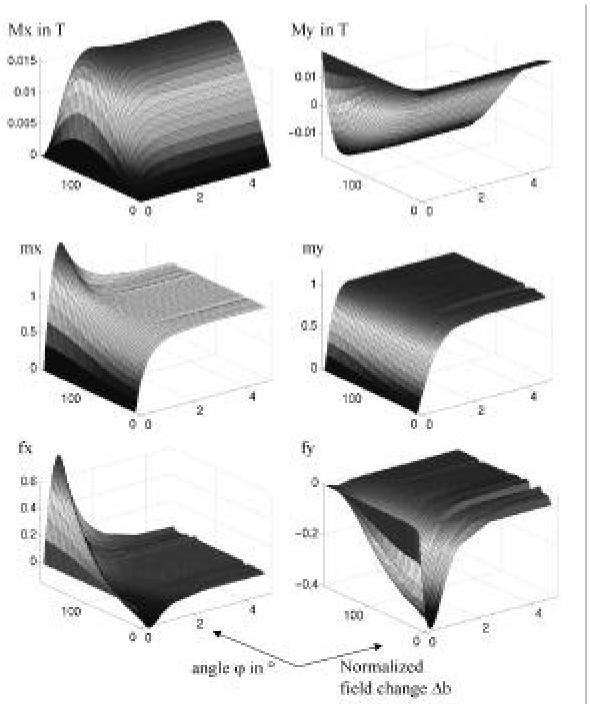

Fig. 2. The magnetization components M_X and M_Y, the normalized components m_X and m_Y and the 'error functions' f_X and f_Y are shown as surface plots.

f_X and f_Y can be described by Eq. (11) with constant exponents a=g=1 and b=11.24. A, b_S and Db_0 are developed as third order polynomes

$$p(j_r) = \sum_{n=0}^{3} p_n j_r^{\,n} \qquad (12)$$

in $j_r = j/360°$. The coefficients for A, b_S and Db_0 are shown in table 1.

f_X	p_0	p_1	p_2	p_3
A	61.3	-177	131	106
b_S	4.20	-8.15	23.9	1.22
Db_0	-0.159	5.53	-43.6	143

f_Y	p_0	p_1	p_2	p_3
A	19.5	90.7	-156	-203
b_S	5.03	-13.6	54.6	-68.4
Db_0	-1.88	6.94	-13.5	16.9

Table 1: Fitting parameters.

2.2. Accuracy

Inaccuracies appear due to a limited creep exponent and a limited mesh discretization. The creep exponent of 500 contributes with less than 0.5%. The grid of 40×40 points especially affects the calculations for angles j of 45° and 135°, where the field change is diagonal to the grid. For angles j with small values of sin(j) or (1-cos(j)) the normalized data is less accurate. The inaccuracies in M_X and M_Y are less than 2% of M_0. The parameterized fitting formula in Eq. (11) is accurate within 3% of M_0.

3. CONCLUSION

We have studied the impact of an external field change with arbitrary direction on the magnetization of a saturated filament. Both, the change of current distribution inside the filament and magnetization are demonstrated in this paper. Fitting formulas are developed to describe the magnetization and parameters are adapted to calculated data.

4. ACKNOWLEDGEMENTS

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